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UNIFIED DARK MATTER AND DARK ENERGY DESCRIPTION IN A CHIRAL COSMOLOGICAL MODEL

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We show the way of dark matter and dark energy presentation via ansatzs on the kinetic energies of the fields in the two-component chiral cosmological model. To connect a kinetic interaction of dark matter and dark energy with observational data the reconstruction procedure for the chiral metric component h_{22} and the potential of (self)interaction V has been developed. The reconstruction of h_{22} and V for the early and later inflation have been performed. The proposed model is confronted to Λ CDM model as well.

Keywords: Chiral cosmological model; cosmic acceleration; dark energy; dark matter.

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1. Introduction

The later-time cosmic acceleration of our Universe is strongly supported by observational data. Namely observations of supernovae type Ia¹, the data from Baryon Acoustic Oscillations (BAO)² and Cosmic Microwave Background (CMB)³ measurements confirm that the Universe is expanding with an acceleration at the present time and about 70% of the energy density consists of dark energy in a wide sense⁴, i.e. as the substance which is responsible for an anti-gravity force.

In the range with well-known Λ CDM model, which potentially provides correct description of the Universe evolution but suffers from fine-tuning and coincidence

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problems, some alternative models were proposed. We will pay attention to the models with presence of scalar fields included in quintessence, phantom and quintom^{5,6,7,8} models.

A chiral cosmological model (CCM) as a nonlinear sigma model with a potential of (self)interactions⁹ has been already used extensively in various areas of gravitation and cosmology^{10,11,12} and in particular for description of the very early Universe^{13,14} and inflation^{15,16}. A CCM can be applicable as well to the late-time Universe with dark matter and dark energy domination as it was shown in¹⁷.

The purpose of this article is to put into use the two-component CCM as the model where the dark energy content of the Universe and also the dark matter component are represented by two chiral fields with kinetic and potential interactions⁹. By considering a target space metric in the form

$$ds_\sigma^2 = h_{11}d\varphi^2 + h_{22}(\varphi, \chi)d\chi^2, \quad h_{11} = \text{const.} \quad (1.1)$$

we prescribe a kinetic interaction between chiral fields φ and χ as a functional dependence h_{22} on the fields. The potential interaction will be included into standard potential energy term of the action.

There are no enough indications from observations about kinetic interactions between dark sector fields. Therefore we always deal with the problem: what is the functional dependence for the chiral metric component on the fields? First idea is to attract some results from HEP, for example, to consider SO(3) symmetry (by taking $h_{22} = \sin^2 \varphi$) and/or others symmetries for a chiral space. From the other hand one can use some testing kinetic interactions^{12,17}.

Thus we can state that there is no evidence for some preferable functional form of the kinetic interaction contained in the functional form of the h_{22} chiral metric component. To avoid this problem we develop here the reconstruction procedure for the chiral metric component h_{22} . We ascribe a certain desirable behavior on the kinetic energy of the second chiral field χ and it becomes possible to determine both the target space metric component h_{22} and a (self)interacting potential V depending on the first chiral field φ . So we can restore a functional dependence the h_{22} and V on the scalar field φ using observational data. Unfortunately it turns out that the procedure could not be applied for the entirely Universe evolution and we have necessity to consider separately the early and late epochs of the Universe evolution.

It will be shown also that a CCM describes dark energy and dark matter in the unified form under special restrictions on the chiral fields (ansatzs). Therefore to include into consideration the present Universe with accelerated expansion it needs to take into account baryonic matter and radiation in the range with a two-component CCM.

Making confrontation of proposed model predictions with observational data we found the way of a reconstruction of a kinetic interaction term h_{22} and the potential V in an exact form. This reconstruction is based on the procedure of finding the best-fit values matching to the astrophysical observations.

The structure of the article is like follow. In section 2, we give the basic model equations and discuss their properties including the exact solutions for a pure CCM (without matter and radiation). We derive the Friedmann equation for the proposed model with the aim to make comparison with Λ CDM in section 3. In section 4, we give the details of a fitting procedure outline. We present the way of the reconstruction of the kinetic coupling and potential in section 4. The early and recent Universe approximations are discussed there as well. Section 6 is devoted to the background dynamics of a CCM. Finally in section 7, we discuss the obtained results and consider perspectives for the future investigations.

2. The model equations and their properties

Recently we proposed a CCM coupling to a perfect fluid¹⁸ with the aim to investigate chiral fields interaction with CDM. For the sake of shortness we termed this model as σ CDM to stress its difference from Λ CDM, Q CDM and others models. σ CDM model presents a generalization of a single scalar field model coupled to CDM in the form of a perfect fluid¹⁹. The model is described by the action functional

$$S = \int d^4x \sqrt{-g} \left(-\frac{1}{2} g^{\mu\nu} h_{AB} \partial_\mu \varphi^A \partial_\nu \varphi^B - V(\varphi^C) \right) + S_{(pf)}. \quad (2.1)$$

Here $S_{(pf)}$ stands for the perfect fluid part of the action, $h_{AB} = h_{AB}(\varphi^C)$ are the target space metric components depending on the scalar fields φ^C . The line element of a target (chiral) space is

$$ds_\sigma^2 = h_{AB}(\varphi^C) d\varphi^A d\varphi^B. \quad (2.2)$$

We use shortened notations for the partial derivatives with respect to the space-time coordinates: $\frac{\partial \varphi^A}{\partial x^\mu} = \partial_\mu \varphi^A$. As usual $g_{\mu\nu}(x^\alpha)$ denotes a space-time metric as a function on the space-time coordinates, so Greek indices α, μ, \dots vary in a range from 0 to 3, Latin capital letters A, B, \dots – take values from 1 to N where N is evidently corresponding to the chiral fields number.

The space-time of homogeneous and isotropic Universe is described by a spatially-flat Friedmann – Robertson – Walker (FRW) metric

$$ds^2 = -dt^2 + a^2(t) (dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)). \quad (2.3)$$

The two-component CCM has a target space metric simplified to¹⁸

$$ds_\sigma^2 = h_{11} d\varphi^2 + h_{22}(\varphi) d\chi^2, \quad h_{11} = \text{const}. \quad (2.4)$$

The σ CDM (2.1) with internal space metric (2.4) includes the models proposed earlier: cold dark matter and cosmological constant (Λ CDM, when $h_{11} = h_{22} = 0$, $V = \text{const} = \Lambda$) model^{5,6}, quintessence model (Q CDM, when $h_{11} = 1$, $h_{22} = 0$), phantom model (Ph CDM, when $h_{11} = -1$, $h_{22} = 0$), quintom model (q CDM, when

4 *R. R. Abbyazov, S. V. Chervon*

$h_{11} = 1, h_{22} = -1$)^{8,20,21,22}. Thus the model under consideration is a generalization of the models investigated earlier and mentioned above.

As a first step of our study we consider the system of equations of the two-component CCM without a perfect fluid. Using assumptions $h_{11} = \text{const}$ and $h_{22} = h_{22}(\varphi)$ expressed in (2.4) one can obtain the system of Einstein and chiral field equations

$$H^2 = \frac{8\pi G}{3} \left[\frac{1}{2} h_{11} \dot{\varphi}^2 + \frac{1}{2} h_{22} \dot{\chi}^2 + V(\varphi, \chi) \right], \quad (2.5)$$

$$\dot{H} = -8\pi G \left[\frac{1}{2} h_{11} \dot{\varphi}^2 + \frac{1}{2} h_{22} \dot{\chi}^2 \right], \quad (2.6)$$

$$\ddot{\varphi} + 3H\dot{\varphi} - \frac{1}{2h_{11}} \frac{dh_{22}}{d\varphi} \dot{\chi}^2 + \frac{1}{h_{11}} \frac{\partial V}{\partial \varphi} = 0, \quad (2.7)$$

$$\ddot{\chi} + 3H\dot{\chi} + \frac{1}{h_{22}} \frac{dh_{22}}{d\varphi} \dot{\varphi} \dot{\chi} + \frac{1}{h_{22}} \frac{\partial V}{\partial \chi} = 0. \quad (2.8)$$

Here $H = \frac{\dot{a}}{a}$, $(\dot{}) = \frac{d}{dt}$.

When first inflationary models were analyzed it was much attention to a very simple case when an inflationary potential $V(\phi)$ equals to the constant²³. Moreover this regime is very important because it leads to an exponential expansion of the Universe. Note that a scalar field is equal to a constant value as well in this regime.

Let us consider for a minute the case of $V = \text{const}$ for the model under consideration (2.5)-(2.8). From (2.8) one can obtain^{16,24}

$$\dot{\chi}^2 = \frac{2C}{h_{22}^2 a^6}. \quad (2.9)$$

Combining (2.5) and (2.6) one can obtain the well-known solution of a de Sitter Universe with Hubble parameter and scale factor⁹

$$H = \sqrt{\frac{\Lambda}{3}} \tanh(\sqrt{3\Lambda}t), \quad a = a_* [\cosh(\sqrt{3\Lambda}t)]^{1/3}. \quad (2.10)$$

This solution with some approximation corresponds to the inflationary stage of the Universe evolution. But our intention is to proceed further in time therefore we need to include into consideration radiation and matter to describe the present epoch of the Universe.

The method of the exact solutions construction for a CCM (2.5)-(2.8) is based on exploiting an additional degree of freedom (see, for ex. discussion in¹⁴). Namely even we fix the potential $V(\phi, \chi)$ there are still four equations with four unknown functions H, φ, χ, h_{22} (h_{11} can be set equal to ± 1 without the loss of generality⁹). Nevertheless the equation (2.5) can be obtained from the linear combination of the chiral field equations (2.7)-(2.8), so the equation (2.5) doesn't independent one. Therefore one may insert the symmetry on the target space or can suggest a testing

interaction between chiral (dark sector) fields^{12,25}. Essentially new approach to this issue we propose here as a reconstruction both h_{22} and V from observational data.

Let us remind that for the scalar field cosmology by introducing the selfinteracting potential $V(\phi)$ we have two equations with two unknown functions. (The same situation will be if we set the dependence a scalar field on time or if we know the scale factor of the Universe as a function on time¹⁴).

To solve the system of a CCM interacting with a perfect fluid (or matter) in explicit form is a very difficult task. Therefore we will use an additional freedom connecting with the chiral metric components h_{11} and h_{22} as a part of a kinetic energy.

An interesting approach for a two-fields model with a cross interaction was proposed in the work¹⁹. To describe a dark matter component it was constructed the special ansatzs for the time derivatives of the scalar fields. In our approach we will use instead some constraints on the kinetic parts of the chiral fields (ansatzs) to obtain a correct description of the present Universe.

Now let us turn our attention to a study of the model equations (2.5)-(2.8). It is easy to check that the solution (2.9) for the constant potential will be valid for the case when $V = V(\varphi)$ only. By extracting from (2.9) the kinetic energy term for the field χ one can obtain

$$\frac{1}{2}h_{22}\dot{\chi}^2 = \frac{C}{h_{22}a^6}. \quad (2.11)$$

We can ascribe by suggestion $h_{22} \sim a^{-3}$ dust matter like behavior to the kinetic energy of the field χ . Using the behavior $h_{22} \sim a^{-3}$ it is easy to see that the second field can be related to the dark matter term provided the restriction to the kinetic energy of the second field χ (ansatz)

$$\frac{1}{2}h_{22}\dot{\chi}^2 = Ca^{-3}. \quad (2.12)$$

Let us mention here, that more simple ansatz $\frac{1}{2}h_{22}\dot{\chi}^2 = \Lambda_\psi = const$ has been analyzed in²⁶ and gave possibility to obtain the exact solutions for the two-component CCM. For the kinetic energy of the first field φ we can form the ansatz by a simple way

$$\frac{1}{2}h_{11}\dot{\varphi}^2 = B = const. \quad (2.13)$$

Further we will show that this relation is associated with the dark energy component in the present Universe.

For convenience let us represent the ansatzs (2.11), (2.13) in the general forms:

$$\frac{1}{2}h_{11}\dot{\varphi}^2 = f(a), \quad (2.14)$$

$$\frac{1}{2}h_{22}\dot{\chi}^2 = g(a). \quad (2.15)$$

Thus we have $f(a) = B = \text{const}$, $g(a) = Ca^{-3}$ in (2.14)-(2.15) and the chiral metric component

$$h_{22} = a^{-3}. \quad (2.16)$$

Let us note that the suggested restrictions above give rise to the exact solution for the CCM describing by equations (2.5)-(2.8). Indeed from ansatzs we can find the solutions for the chiral fields

$$\varphi = \sqrt{\frac{2B}{h_{11}}}t + \varphi_0, \quad \chi = \sqrt{2C}t + \chi_0. \quad (2.17)$$

Then from Einstein equations (2.5)-(2.6) we can define the potential

$$V(a) = -6B \ln a + Ca^{-3} + V_*. \quad (2.18)$$

The solution for the scale factor can be obtained from the equation

$$H^2 = \frac{C_*}{a^6} + 2\kappa \left(\frac{B}{6} - B \ln a + \frac{C}{3a^3} + \frac{V_*}{6} \right).$$

It is difficult to find the scale factor in exact view from this general equation, but for the special case assuming $C_* = 0$ and $C = 0$ (under this assumption the second field χ becomes a constant), we found that the Universe is in the stage with an exponential expansion with $a \propto \exp(Bt^2)$.

3. A CCM coupling to barion matter and radiation. Friedmann equation of the model

Our following task is to connect the energy densities of various species of the Universe to the Hubble parameter. To this end we need to include into Friedmann equation (2.5) the energy density of barion matter ρ_b and radiation ρ_r . Thus (2.5) for the recent Universe takes the form

$$H^2 = \frac{8\pi G}{3} [\rho_\sigma + \rho_b + \rho_r] \quad (3.1)$$

where $\rho_\sigma = \frac{1}{2}h_{11}\dot{\varphi}^2 + \frac{1}{2}h_{22}\dot{\chi}^2 + V$. Introducing the "pressure" of chiral fields $p_\sigma = \frac{1}{2}h_{11}\dot{\varphi}^2 + \frac{1}{2}h_{22}\dot{\chi}^2 - V$ and using ansatzs (2.14) and (2.15) we can obtain

$$\rho_\sigma = f + g + V, \quad p_\sigma = f + g - V.$$

Using (2.18) and extracting the cosmological parameter Λ from V_* the energy density, potential and pressure of the two-component CCM can be expressed as

$$\rho_\sigma = \Lambda - 6B \ln a + 2Ca^{-3}, \quad \Lambda = B + V_* \quad (3.2)$$

$$V = \Lambda - 6B \ln a + Ca^{-3} - B, \quad (3.3)$$

$$p_\sigma = 2B - \Lambda + 6B \ln a. \quad (3.4)$$

By standard way (see, for ex. ²⁷) one can define a critical density $\rho_c = \frac{3H_0^2}{8\pi G}$, where H_0 is the Hubble parameter of today expansion $H_0 = \dot{a}(t_0)$. Herefrom the subscript "0" is related to the present time t_0 when the scale factor $a(t_0) = a_0 = 1$. Also we will use the density parameter $\Omega_0 = \frac{\rho}{\rho_c}(t_0)$ and the individual ratios $\Omega_i = \frac{\rho_i}{\rho_c}(t_0)$ for chiral fields, barion matter and radiation.

Let us remember that equations of state for radiation and baryons are

$$p_r = \frac{1}{3}\rho_r, \quad p_b = 0.$$

The energy densities and the contribution to the critical density can be represented as

$$\rho_r = \rho_{r0}a^{-4} = \Omega_{r0}\rho_{c0}a^{-4}, \quad \rho_b = \rho_{b0}a^{-3} = \Omega_{b0}\rho_{c0}a^{-3}, \quad \rho_{c0} = \frac{3H_0^2}{8\pi G}.$$

Taking into account (3.2) Friedmann equation (3.1) can be transformed to the normalised Hubble parameter form

$$\frac{H^2}{H_0^2} = \frac{1}{\rho_c} (\Lambda - 6B \ln a + 2Ca^{-3}) + \Omega_{b0}a^{-3} + \Omega_{r0}a^{-4}.$$

Making renormalization of the constants we finally obtain the normalised Hubble rate in the form which is suitable for further confronting with observational data

$$\tilde{H}^2 = \frac{H^2}{H_0^2} = \tilde{\Lambda} - 6\tilde{B} \ln a + 2\tilde{C}a^{-3} + \Omega_{b0}a^{-3} + \Omega_{r0}a^{-4}, \quad (3.5)$$

where

$$\tilde{B} = \frac{B}{\rho_c}, \quad \tilde{C} = \frac{C}{\rho_c}, \quad \tilde{\Lambda} = \frac{\Lambda}{\rho_c}, \quad \tilde{H}^2 = \frac{H^2}{H_0^2}. \quad (3.6)$$

We need to find $\tilde{\Lambda}$ at $a = a_0 = 1$ with the help of Friedmann equation. Cold dark matter (CDM) is included in the model as the kinetic ansatz (2.11)

$$\tilde{\Lambda} = 1 - 2\tilde{C} - \Omega_{b0} - \Omega_{r0} = \Omega_{\sigma\Lambda 0}, \quad \Omega_{\sigma cdm 0} = 2\tilde{C}, \quad \Omega_{m0} = \Omega_{\sigma cdm 0} + \Omega_{b0}. \quad (3.7)$$

Summing up the notations above we display the final form of the normalised Hubble parameter

$$\tilde{H}^2 = \Omega_{\sigma\Lambda 0} - 6\tilde{B} \ln a + \Omega_{\sigma cdm 0}a^{-3} + \Omega_{b0}a^{-3} + \Omega_{r0}a^{-4}. \quad (3.8)$$

We propose here the σ CDM model containing the dark energy with variable equation of state. The model is an alternative to Λ CDM model with the cosmological constant and CDM. Let us note that generally speaking the presence of \tilde{B} in (3.8) may change the values of Ω_{m0} and $\Omega_{\Lambda 0}$, thus they can be distinctive from the corresponding quantities in Λ CDM model. Nevertheless to find the exact values of this distinction we need to perform comparison with the experimental data.

4. Comparison with experimental data

From the very beginning^{28, 29} supernovae Ia type observations directly indicated an accelerated expansion of the Universe. Observing supernovae luminosity distance d_L as a function of a redshift one can infer about an expansion history of the Universe. Here we use one of the most recent compilation of the supernovae sets Union 2.1¹. The procedure of confronting cosmological model predictions with observations consists of minimizing quantity procedure and calculation as a result best-fit values of the model parameters for

$$\chi_{SN}^2 = \sum_{i=1}^N \frac{[\mu_{obs}(z_i) - \mu(z_i)]^2}{\sigma_i^2(z_i)}. \quad (4.1)$$

Here as usual in the supernovae experimental analysis³⁰ module distance $\mu(z_i)$ is used. The dependence on the luminosity distance is

$$\mu(z_i) = 5 \log_{10}[D_L(z_i)] + \mu_0, \quad D_L = H_0 d_L, \quad \tilde{H} = H/H_0. \quad (4.2)$$

$$\mu_0 = 5 \log_{10} \left[\frac{H_0^{-1}}{Mpc} \right] + 25 = 42.38 - 5 \log_{10} h, \quad H_0 = \frac{h}{2998} \text{Mpc}^{-1}. \quad (4.3)$$

In order to find more accurate parameter values and to reduce errors significantly it is necessary to supplement the supernovae observations with information about baryonic acoustic oscillations (BAO)² and cosmic microwave background (CMB)³.

BAO χ^2 function is defined as

$$\chi_{BAO}^2 = \left(\frac{D_V(z=0.35)/D_V(z=0.2) - 1.736}{0.065} \right)^2, \quad (4.4)$$

where

$$D_V \equiv \left[(1+z)^2 D_A^2(z) \frac{z}{H(z)} \right]^{1/3} \quad (4.5)$$

is an effective distance measure, while

$$D_A = (1+z)^{-2} d_L(z) \quad (4.6)$$

is the angular diameter distance².

Function χ^2 for CMB is

$$\chi_{CMB}^2 = (x_i^{th} - x_i^{obs})(C^{-1})_{ij}(x_j^{th} - x_j^{obs}), \quad (4.7)$$

where $x_i = (l_A, R, z_*)$ — the vector of quantities which characterizes the cosmological model and $(C^{-1})_{ij}$ — WMAP7 covariance matrix³. Here we use acoustic scale,

from which first acoustic peak of CMB power spectrum is depending on ⁴

$$l_A \equiv (1 + z_*) \frac{\pi D_A(z_*)}{r_s(z_*)}, \quad (4.8)$$

which has been taken at the moment z_* of decoupling of radiation from matter, and on the sound horizon

$$r_s(z) = \frac{1}{\sqrt{3}} \int_0^{1/(1+z)} \frac{da}{a^2 H(a) \sqrt{1 + (3\Omega_b/4\Omega_\gamma)a}}. \quad (4.9)$$

We will use the fitting formula

$$z_* = 1048[1 + 0.00124(\Omega_b h^2)^{-0.738}][1 + g_1(\Omega_m h^2)^{g_2}], \quad (4.10)$$

$$g_1 = \frac{0.0783(\Omega_b h^2)^{-0.238}}{1 + 39.5(\Omega_b h^2)^{0.763}}, \quad g_2 = \frac{0.560}{1 + 21.1(\Omega_b h^2)^{1.81}}, \quad (4.11)$$

for decoupling moment ³¹. Shift parameter R is defined as

$$R(z_*) = \sqrt{\Omega_{m0} H_0^2 (1 + z_*)} D_A(z_*). \quad (4.12)$$

Minimizing the sum $\chi_{joint}^2 = \chi_{SN}^2 + \chi_{BAO}^2 + \chi_{CMB}^2$ one can find the best-fit \tilde{B} and \tilde{C} values. We also keep fixed the radiation and baryonic contributions to the critical density today $\Omega_{\gamma 0} = 2.469 \cdot 10^{-5} h^{-2}$, $\Omega_{b0} = 0.022765 \cdot 10^{-2}$, $h = 0.742$. Also we take into account a relativistic neutrino in addition to the photon radiation component $\Omega_{r0} = (1 + N_{eff})\Omega_{\gamma 0}$, where $N_{eff} = 3.04$ — the effective neutrino number ³⁰. Our results for the best-fit from χ_{joint}^2 minimization are $\tilde{B} = 0.00078$, $\Omega_{\sigma m0} = \Omega_{b0} + 2\tilde{C} = \Omega_{b0} + \Omega_{\sigma cdm0} = 0.23398$. For Λ CDM model we take best-fit values $\Omega_{m0} = 0.27$ and $\Omega_{\Lambda 0} = 1 - \Omega_{m0} - \Omega_{r0}$. To avoid confusion between Ω_{cdm} and Ω_m in σ CDM and Λ CDM models we put additional index σ in Ω above.

5. The reconstruction of the metric component h_{22} and the potential V

In order to learn more about kinetic and potential interactions between DM and DE we must extend the standard reconstruction of the expansion history of the Universe ³² to a restoration of a functional dependence on the scalar field φ for the target space metric component $h_{22} = h_{22}(\varphi)$ and the potential $V = V(\varphi)$ of σ CDM.

Let us transform the ansatz (2.14) (setting $h_{11} = 1$)

$$\frac{1}{2} \dot{\varphi}^2 = B = f, \quad \frac{1}{2} \left(\frac{d\varphi}{dt} \right)^2 = B, \quad (5.1)$$

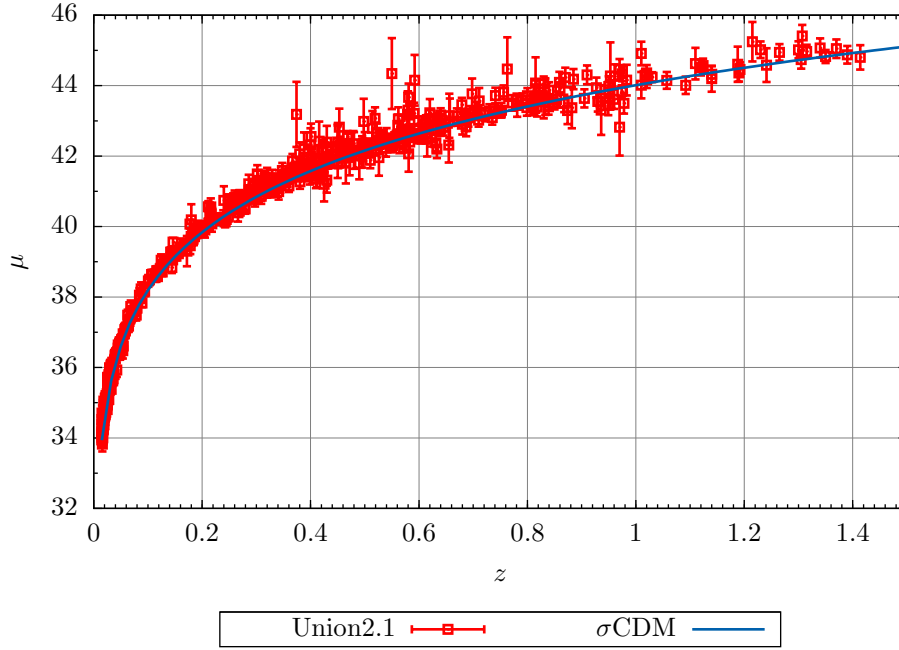


Fig. 1. Supernovae Union 2.1 data and prediction from σCDM model.

Changing variables from t to a

$$\frac{1}{2} \left(\frac{d\varphi}{da} \right)^2 = \left(\frac{dt}{da} \right)^2 B, \quad (5.2)$$

and introducing already known for us (3.8) Hubble parameter one can obtain

$$\frac{1}{2} \varphi'^2 = \frac{B}{H_0^2 a^2 \tilde{H}^2}. \quad (5.3)$$

Our next goal is to find the dependence $\varphi = \varphi(a)$, so we fix limits of integration from some early epoch a_i up to desired time moment, corresponded to a

$$\int_{a_i}^a H_0 \varphi' da = \sqrt{2} \int_{a_i}^a \frac{\sqrt{B} da}{a \tilde{H}}. \quad (5.4)$$

The integral written here cannot be taken in an explicit form. Nevertheless there is a possibility to use some approximations based on a behavior of the different energy densities components with a scale factor. Therefore following the idea of ¹⁹ we consider the early $a \ll 1$ and recent Universe $a \approx 1$ approximations.

Let us start from the case $a \ll 1$. The Universe is known to be radiation dominated at the very early times. This means that all components contribution in Hubble parameter is negligible in comparison with the radiation term. So such observation makes possible to do integration in

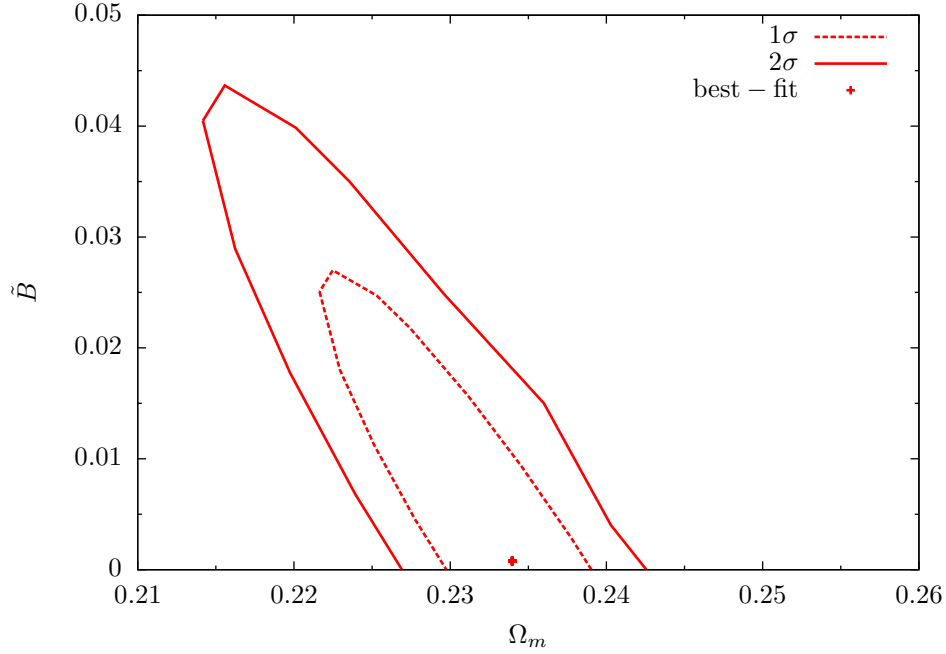


Fig. 2. Contour plots corresponding to 1σ (68%) 1σ and 2σ (95%) likelihood levels for σ CDM model parameters.

$$H_0 (\varphi(a) - \varphi(a_i)) = \int_{a_i}^a \frac{\sqrt{2B}}{a \sqrt{\frac{\Omega_{r0}}{a^4}}} da = \frac{\sqrt{B}}{\sqrt{2\Omega_{r0}}} (a^2 - a_i^2).$$

We can normalize the scalar field on today's critical density $\varphi \rightarrow \frac{\varphi}{\sqrt{\rho_{c0}}}$, which gives us \tilde{B} instead of B in (5.3). Here $\varphi(a_i) = \varphi(a = a_i)$, where a_i is the fixed value of a scale factor which will be taken equal to 10^{-5} and normalized to current value a_0 . It will be helpful for our analysis to introduce $\tilde{\varphi}_{early} = \varphi(a_i) - \frac{\sqrt{\tilde{B}}}{H_0 \sqrt{2\Omega_{r0}}} a_i^2 = \varphi(a_i) + const$ and $\varphi_{early} = \varphi(a_i)$.

Now we have to invert $\varphi = \varphi(a)$ dependence

$$H_0 (\varphi - \tilde{\varphi}_{early}) = \frac{\sqrt{\tilde{B}}}{\sqrt{2\Omega_{r0}}} a^2$$

to get the $a = a(\varphi)$ dependence

$$a = \sqrt{\frac{H_0 \sqrt{2\Omega_{r0}}}{\sqrt{\tilde{B}}} (\varphi - \tilde{\varphi}_{early})}.$$

If we know a we can write down the chiral metric component h_{22} as a function on φ

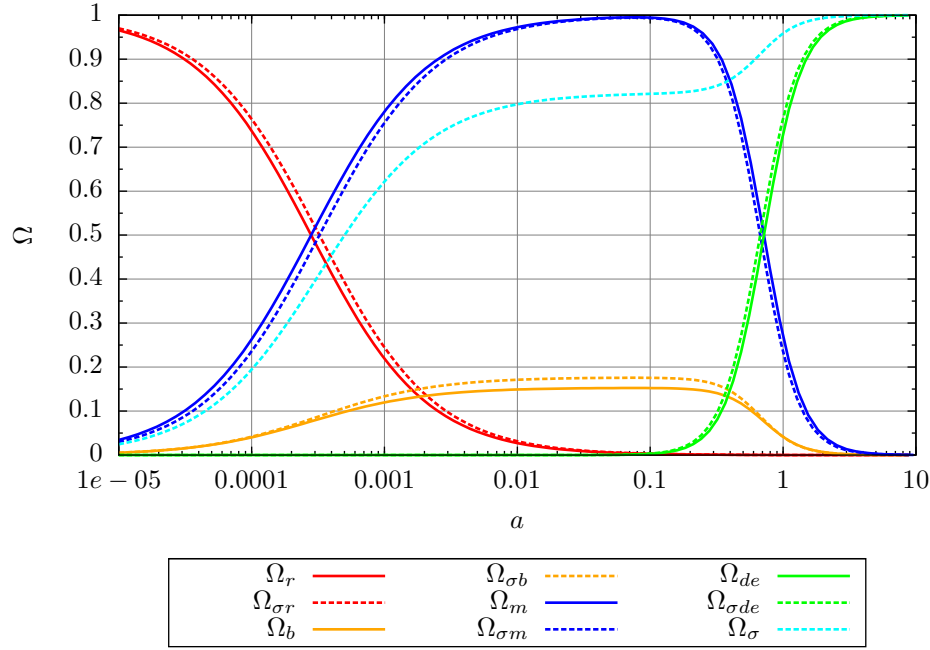


Fig. 3. Evolution of contributions to critical density of various components in Λ CDM and σ CDM models

$$h_{22} = a^{-3} = \left(\frac{H_0 \sqrt{2\Omega_{r0}}}{\sqrt{\tilde{B}}} (\varphi - \tilde{\varphi}_{early}) \right)^{-3/2}$$

Taking the constant $V_0 = \Lambda - B$ in (3.3) one can obtain

$$V = V_0 - 3B \ln \left[\frac{H_0 \sqrt{2\Omega_{r0}}}{\sqrt{\tilde{B}}} (\varphi - \tilde{\varphi}_{early}) \right] + C \left[\frac{H_0 \sqrt{2\Omega_{r0}}}{\sqrt{\tilde{B}}} (\varphi - \tilde{\varphi}_{early}) \right]^{-3/2}.$$

Thus we have finished the procedure of reconstruction of h_{22} and V for the very early epoch of the Universe evolution when $a \ll 1$.

Next step in consideration is the recent Universe approximation with $a \approx 1$. Transforming the Hubble parameter to the form (3.8)

$$\tilde{H}^2(a) = 1 + \frac{\Omega_{r0}}{a^4} (1 - a^4) + \frac{\Omega_{m0}}{a^3} (1 - a^3) - 6\tilde{B} \ln a,$$

we obtain

$$\tilde{H}^2(a) = 1 + \frac{\Omega_{r0}}{(1 - (1 - a))^4} - \Omega_{r0} + \frac{\Omega_{0m}}{(1 - (1 - a))^3} - \Omega_{0m} - 6\tilde{B} \ln(1 - (1 - a)). \quad (5.5)$$

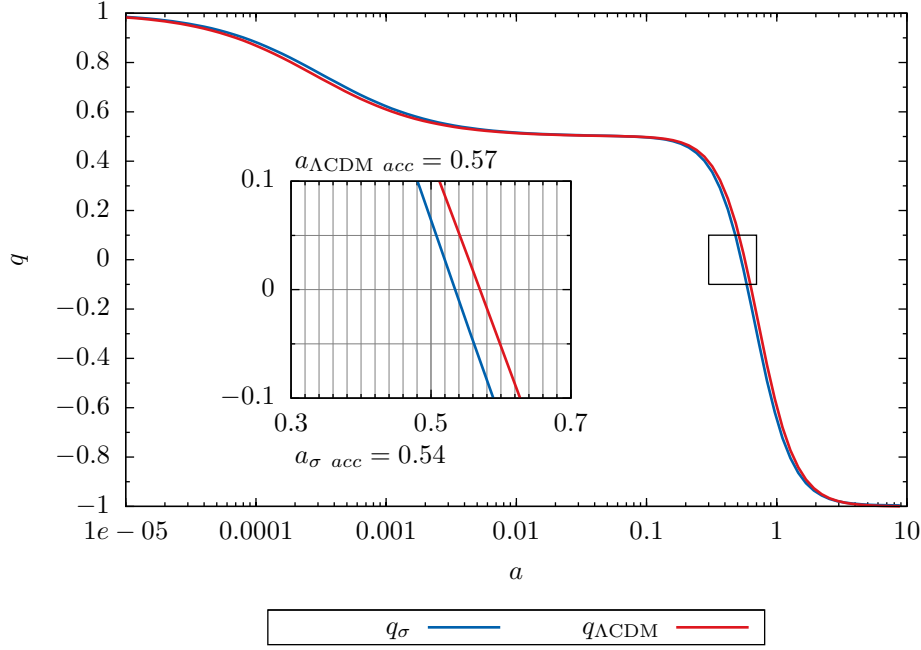


Fig. 4. Evolution of the deceleration parameter in Λ CDM and σ CDM models

Let us apply the Taylor expansion about $(1-a) \approx 0$ up to first order terms in (5.5). The result is

$$\tilde{H}^2(a) = 1 + \left(4\Omega_{r0} + 3\Omega_{\sigma m0} + 6\tilde{B}\right)(1-a), \quad \Omega_{\sigma m0} = \Omega_{b0} + \Omega_{\sigma cdm0}. \quad (5.6)$$

From this moment we are able to fulfill the reconstruction procedure. Dividing scalar field on $\sqrt{\rho_{c0}}$ once again we come to

$$H_0(\varphi(a) - \varphi(a_i)) = \int_{a_i}^a \frac{\sqrt{2\tilde{B}}da}{a\sqrt{1 + \left(3\Omega_{0m} + 4\Omega_r + 6\tilde{B}\right)(1-a)}} = \int_{a_i}^a \frac{\sqrt{2\tilde{B}}da}{a\sqrt{\alpha - \beta a}}, \quad (5.7)$$

where

$$\beta = 3\Omega_{m0} + 4\Omega_{r0} + 6\tilde{B} = 3 \cdot 0.23 + 4 \cdot 5 \cdot 10^{-5} \cdot (1 + 0.6) + 6 \cdot 0.007 > 0,$$

$$\alpha = 1 + \beta > 1.$$

It is essential for the subsequent analysis that the best-fit value of \tilde{B} is known, so we have an opportunity to make calculation of the integral (5.7). Such a type of an integral is calculated by

$$\int \frac{dx}{\sqrt{x(x-b)}} = -\frac{1}{\sqrt{b}} \ln \left| \frac{\sqrt{x} + \sqrt{b}}{\sqrt{x} - \sqrt{b}} \right|, \quad b > 0.$$

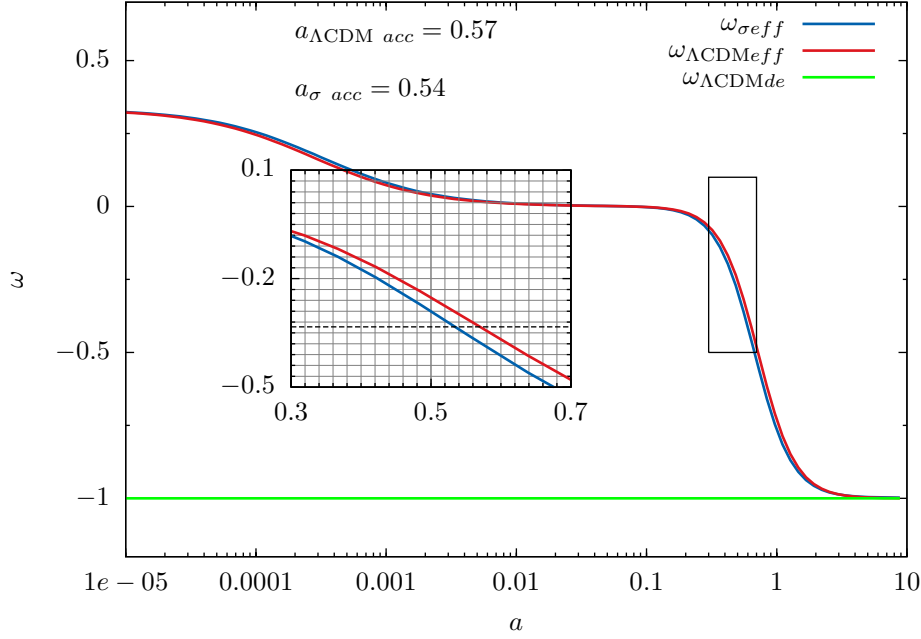


Fig. 5. Evolution of the effective equation of state parameter in Λ CDM and σ CDM models

One more issue is about a transition through $a = 1$. This scale factor value should be explicitly presented in the expression for φ

$$H_0(\varphi - \varphi(a = a_i)) = \sqrt{2\tilde{B}} \left\{ \int_{a_i}^1 \frac{da}{a\tilde{H}} + \int_1^a \frac{da}{a\tilde{H}} \right\}. \quad (5.8)$$

With the help of

$$\tilde{\varphi}_{recent} = \varphi(a = a_i) + \frac{\sqrt{2\tilde{B}}}{H_0} \frac{1}{\sqrt{\alpha}} \ln \left| \frac{\sqrt{\alpha - \beta a_i} + \sqrt{\alpha}}{\sqrt{\alpha - \beta a_i} - \sqrt{\alpha}} \right|,$$

and

$$\varphi_{recent} = \varphi(a = a_i) + \frac{\sqrt{2\tilde{B}}}{H_0} \frac{1}{\sqrt{\alpha}} \left[-\ln \left| \frac{\sqrt{\alpha - \beta} + \sqrt{\alpha}}{\sqrt{\alpha - \beta} - \sqrt{\alpha}} \right| + \ln \left| \frac{\sqrt{\alpha - \beta a_i} + \sqrt{\alpha}}{\sqrt{\alpha - \beta a_i} - \sqrt{\alpha}} \right| \right],$$

one can carry out computations further in more compact form. Plausibility of the early and recent approximations can be deduced from the comparison of $H_0(\varphi - \varphi_{early})$ and $H_0(\varphi - \varphi_{recent})$ for the exact (3.8) and approximate (5.6) Hubble parameter expressions and the time limits (for which corresponding approximations are hold on) will be extracted graphically.

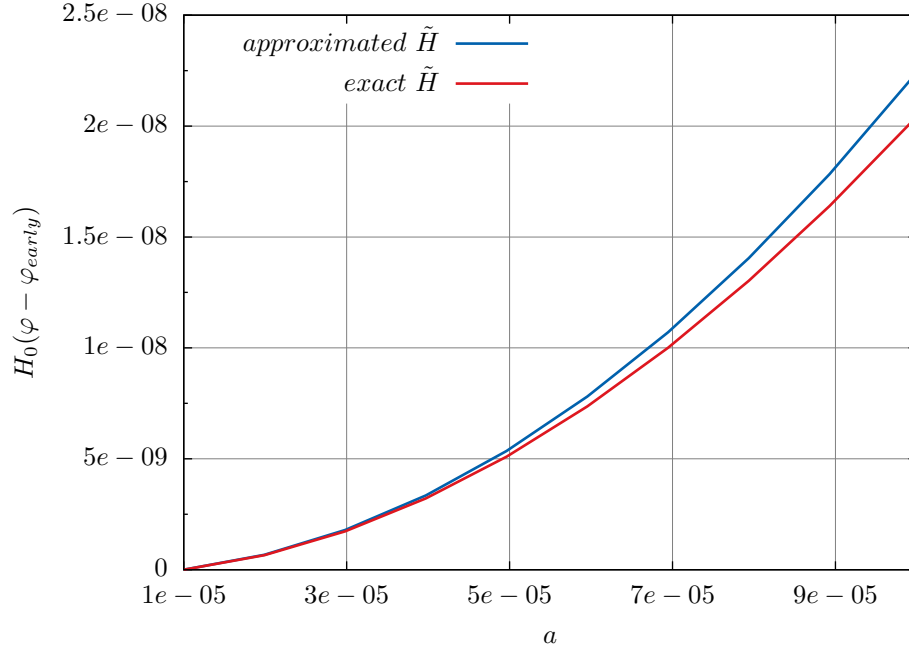


Fig. 6. Early Universe approximation.

The reconstruction is performed as usual when $a = a(\varphi)$ is obtained. Using notation of the $\tilde{\varphi}_{recent}$ in (5.8) we have

$$H_0(\varphi - \tilde{\varphi}_{recent}) = -\frac{\sqrt{2\tilde{B}}}{\sqrt{\alpha}} \ln \left| \frac{\sqrt{\alpha - \beta a} + \sqrt{\alpha}}{\sqrt{\alpha - \beta a} - \sqrt{\alpha}} \right|,$$

and

$$a = \frac{\alpha}{(\alpha - 1) \cosh^2(A(\varphi))}, \quad A(\varphi) = -\frac{\sqrt{\alpha}}{2\sqrt{2\tilde{B}}} H_0(\varphi - \tilde{\varphi}_{recent}).$$

We can substitute this result to h_{22} (2.16) and V (3.3) to get

$$h_{22} = \left(\frac{\alpha - 1}{\alpha} \right)^3 \cosh^{-6}(A(\varphi)),$$

$$V = V_0 - 6B \ln \left[\frac{\alpha}{(\alpha - 1) \cosh^2(A(\varphi))} \right] + C \left[\frac{\alpha}{(\alpha - 1) \cosh^2(A(\varphi))} \right]^{-3}.$$

6. Background dynamics of the model

One of the most important cosmological parameter used for the description of a background evolution is a contribution to a critical density of the Universe. The

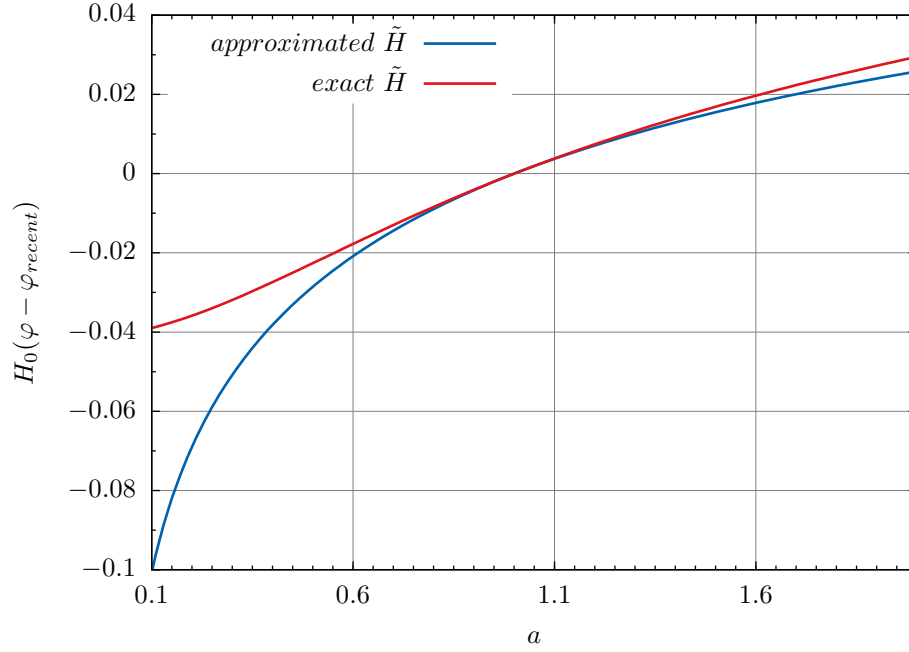


Fig. 7. Recent Universe approximation.

last is defined as $\Omega = \frac{\rho}{\rho_c}$. For the chiral fields sector we have

$$\Omega_\sigma = \frac{\rho_\sigma}{\rho_c} = \frac{\rho_\sigma}{\frac{3H_0^2 \tilde{H}^2}{8\pi G}}.$$

Using (3.2) and (3.8) one can obtain

$$\Omega_\sigma = \frac{\tilde{\Lambda} - 6\tilde{B} \ln a + 2\tilde{C}a^{-3}}{\Omega_{\sigma\Lambda 0} + \Omega_{\sigma cdm 0}a^{-3} + \Omega_{b0}a^{-3} + \Omega_{r0}a^{-4} - 6\tilde{B} \ln a}.$$

In order to understand a general picture of the Universe evolution and to analyze periods of domination by various species of the Universe we represent the residual components of σ CDM model

$$\Omega_{\sigma r} = \frac{\Omega_{r0}a^{-4}}{\tilde{H}^2}, \quad \Omega_{\sigma b} = \frac{\Omega_{b0}a^{-3}}{\tilde{H}^2}, \quad \Omega_{\sigma m} = \frac{\Omega_{m0}a^{-3}}{\tilde{H}^2}, \quad \Omega_{\sigma de} = \frac{\Omega_{\sigma\Lambda 0} - 6\tilde{B} \ln a}{\tilde{H}^2},$$

where \tilde{H} comes from (3.8). The latter quantity is responsible for the late accelerated expansion of the Universe, supported by σ CDM model.

In the Λ CDM we have

$$\Omega_r = \frac{\Omega_{r0}a^{-4}}{\tilde{H}^2}, \quad \Omega_b = \frac{\Omega_{b0}a^{-3}}{\tilde{H}^2}, \quad \Omega_m = \frac{\Omega_{m0}a^{-3}}{\tilde{H}^2}, \quad \Omega_{de} = \frac{\Omega_\Lambda}{\tilde{H}^2}.$$

Here \tilde{H}^2 is given by

$$\tilde{H}^2 = \Omega_{\Lambda 0} + \Omega_{m0}a^{-3} + \Omega_{b0}a^{-3} + \Omega_{r0}a^{-4}. \quad (6.1)$$

Let us turn our attention to the effective equation of the state parameter

$$\omega_{eff} = \frac{\sum_{\alpha} p_{\alpha}}{\sum_{\alpha} \rho_{\alpha}}.$$

It is necessary to take into account expressions for densities and pressures of the chiral fields (3.2), (3.4) and the other components, together with (3.8)

$$\rho_r = \rho_{r0}a^{-4} = \Omega_{r0}\rho_{c0}a^{-4}, \quad \rho_b = \rho_{b0}a^{-3} = \Omega_{b0}\rho_{c0}a^{-3}, \quad p_r = \frac{1}{3}\rho_r, \quad p_b = 0.$$

Then in the σ CDM model we will have

$$\omega_{\sigma(eff)} = \frac{p_r + p_b + p_{\sigma}}{\rho_r + \rho_b + \rho_{\sigma}} = \frac{\frac{1}{3}\Omega_{r0}a^{-4} + (-\Omega_{\sigma\Lambda 0} + 6\tilde{B}\ln a + 2\tilde{B})}{\Omega_{r0}a^{-4} + \Omega_{\sigma m0}a^{-3} + \Omega_{\sigma\Lambda 0} - 6\tilde{B}\ln a}.$$

At the same time for Λ CDM model the effective equation of state parameter is

$$\omega_{\Lambda\text{CDM}(eff)} = \frac{1}{2} \frac{2\Omega_{r0}a^{-4} + \Omega_{m0}a^{-3} + (-2\Omega_{\Lambda 0})}{\Omega_{\Lambda 0} + \Omega_{m0}a^{-3} + \Omega_{b0}a^{-3} + \Omega_{r0}a^{-4}}.$$

Using the definition of the deceleration parameter q broadly used for background dynamics studies we can obtain

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = \frac{\frac{4\pi G}{3}(\sum_{\alpha} \rho_{\alpha} + 3p_{\alpha})}{\frac{8\pi G}{3}\sum_{\alpha} \rho_{\alpha}}.$$

Presence in the q the second derivative of the scale factor gives us evidence of the transition from deceleration to acceleration epoch at the time when $q = 0$. The deceleration parameter for chiral sector takes the view

$$q_{\sigma} = \frac{1}{2} \frac{\Omega_{\sigma m0}a^{-3} + 2\Omega_{r0}a^{-4} + 12\tilde{B}\ln a - 2\Omega_{\sigma\Lambda 0} + 6\tilde{B}}{\tilde{H}^2},$$

where \tilde{H}^2 is defined in (3.8). For Λ CDM model the expression for deceleration parameter looks like

$$q_{\Lambda\text{CDM}} = \frac{1}{2} \frac{(\Omega_{b0}a^{-3} + 2\Omega_{r0}a^{-4} + \Omega_{cdm0}a^{-3} - 2\Omega_{\Lambda 0})}{\tilde{H}^2}$$

with Hubble parameter taken from (6.1). The evident differences both in numerator and denominator in q_{σ} and $q_{\Lambda\text{CDM}}$ inevitably lead to distinctive evolution of the Universe if it is supported by σ CDM or Λ CDM models.

It is known feature of ω_{eff} that a moment of time of deceleration/acceleration transition corresponds to value $-1/3$ crossing. It is also well-known result this time to be exactly equivalent to those obtained from q analysis. We would like to point out here that in Λ CDM model equation of state of the dark energy parameter is equal $\omega_{\Lambda\text{CDM}de} = -1$.

7. Discussion

Fig.1 shows good agreement of supernovae data with σ CDM model taken with the best-fit parameters values. This fact confirm the validity of proposed model, i.e., σ CDM model does not contradict to observational data and may serve as a good dynamical alternative to Λ CDM.

In fig. 2 the confidence contours are depicted. We keep only positive values of parameter \tilde{B} in order to prevent a crossing of the phantom divide.

One can see from the evolution of the individual densities Ω_i , deceleration parameters q and effective equation of state parameters ω_{eff} (figs. 3, 4 and 5) that accelerated expansion takes place earlier in the Universe supported by σ CDM. Also one may notice that radiation/matter domination transition occurs earlier in Λ CDM model. These observations are in the full agreement with a smaller total matter amount including cold dark and baryonic components in σ CDM model in comparison to Λ CDM model.

The graphical comparison (see fig. 4 and fig. 5) of the $a_{\Lambda\text{CDM}acc}$ and $a_{\sigma acc}$ (taken from the scale factor values corresponding to $-1/3$ and 0 crossing) gives us clear evidence for equality of the transitions to accelerate expansion in corresponding models. This observation is concluded from q and ω_{eff} values and has been already mentioned above.

From fig. 6 one can conclude that the early Universe approximation holds for $a = 10^{-5}$ up $a = 5 \cdot 10^{-5}$ scale factor values. The recent Universe approximation depicted on fig. 7 is true from $a = 0.8$ to $a = 1.2$ values. Let us remind that validity of the early and recent approximations comes from confrontation of $H_0(\varphi - \varphi_{early})$ and $H_0(\varphi - \varphi_{recent})$. The deviation for approximated and exact \tilde{H} is associated with the lost of domination of DE for the early times.

In conclusion it needs to stress that we first time reconstructed from observations the kinetic interaction between DM and DE in the form of chiral metric component h_{22} for σ CDM. Also we have hope that the reconstruction techniques presented here may be useful for exact solution construction because of obtaining h_{22} from observational data.

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